

# A Characterization of Generalized Complex Ellipsoids In $\mathbb{C}^n$ and Related Results

AKIO KODAMA, STEVEN G. KRANTZ & DAOWEI MA

**Introduction.** Let  $D$  be a bounded domain in  $\mathbb{C}^n$  and  $x$  a point on the boundary  $\partial D$ . Assume that there is a local holomorphic peaking function for  $D$  at  $x$ , and that  $x$  is an accumulation point of an  $\text{Aut}(D)$ -orbit. *Can we then determine the global structure of  $D$  from the local shape of the boundary  $\partial D$  near  $x$ ?* There are several papers closely related to this problem. For instance, Wong [30] and Rosay [26] investigated the case where  $x$  is a  $C^2$  strictly pseudoconvex boundary point; Frankel [9] gave a characterization of bounded symmetric domains among the category of hyperbolic convex domains  $D$  in  $\mathbb{C}^n$  by using the so-called rescaling or blow-up technique at some boundary points. This technique of blow-up was later extended by Kim [14] to the case where  $D$  near  $x$  is locally biholomorphically equivalent to a  $C^\omega$  convex domain. Moreover, a complete characterization of a bounded pseudoconvex domain in  $\mathbb{C}^2$  with  $C^\omega$  boundary and non-compact automorphism group can be found in Bedford and Pinchuk [1]. Bedford and Pinchuk [2] have some generalizations of this result to higher dimensions and to domains of finite type. More recently, Bedford and Pinchuk [3] have given a classification of convex domains with non-compact automorphism group. Also, Greene and Krantz [12, 13], Kodama [16, 17, 18] and Berteloot [4] studied the above problem in the case where  $D$  near  $x$  coincides with the complex ellipsoid

$$E(p_1, \dots, p_n) = \left\{ z \in \mathbb{C}^n; \sum_{i=1}^n |z_i|^{2p_i} < 1 \right\},$$

where  $p_i$  are positive integers. In this paper, we shall study the structure of a generalized complex ellipsoid

$$E(n; n_1, \dots, n_s; p_1, \dots, p_s) = \left\{ (z_1, \dots, z_s) \in \mathbb{C}^{n_1} \times \dots \times \mathbb{C}^{n_s}; \sum_{i=1}^s \|z_i\|^{2p_i} < 1 \right\}$$

in  $\mathbb{C}^n = \mathbb{C}^{n_1} \times \dots \times \mathbb{C}^{n_s}$ , where  $p_1, \dots, p_s$  are positive numbers and  $n_1, \dots, n_s$  are positive integers, and  $n = n_1 + \dots + n_s$ . In general this domain is not geometrically convex and its boundary is not smooth.