

Invariant Proper Holomorphic Maps between Balls

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ABSTRACT. In this paper we consider proper holomorphic maps between balls that are invariant under the action of finite fixed point free unitary groups. Forstnerič showed that given any such group (representation) there exists such a map. He showed that if we also require the map to be smooth to the boundary, then many available groups are ruled out. In this paper we prove the following theorem: *If f is a proper holomorphic map between balls that is smooth to the boundary and invariant under the action of such a group, then necessarily that group is cyclic.* We rule out some of these cases as well. We discuss some related theory concerning group-invariant proper holomorphic maps between balls. One result uses a modification of a technique due to Løw, that embeds smooth strongly pseudoconvex domains properly into balls, to produce new examples of group-invariant maps.

1. Introduction. We will investigate proper maps between balls in complex Euclidean space. Proper holomorphic maps in several complex variables have received a good deal of attention in recent years (see survey articles [CS3] and [Fo3], for example). When domain and range are balls, one can use powerful general results about maps between strongly pseudoconvex domains with real analytic boundaries, such as extension results that arise from polarization and related theory. One also has available techniques related to the inverse of balls under the action of the unitary group. We put such techniques to use in the main result of this paper.

Some research ([D1], [Fo1], [Ru1], [Ru2]) has focused on proper holomorphic maps from balls that are invariant under the action of some finite unitary matrix group acting on the domain. In [Ru2] and [Fo1] it is shown that proper holomorphic maps from balls that are smooth to the boundary factor through finite unitary matrix groups (after perhaps first applying an automorphism on the domain). Rudin assumes domain and range have the same dimension, larger