

Boundedness of Oscillatory Singular Integrals on Hardy Spaces: II

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ABSTRACT. In this paper, we study the uniform boundedness of oscillatory singular integral operators of convolution type on the Hardy space H^1 . We prove that the uniform boundedness holds if the phase function does not vanish of infinite order at the origin. It is also shown that uniform boundedness may fail if such condition is removed.

1. Introduction. In a previous paper [6] it was proved that oscillatory singular integrals with polynomial phase functions are bounded on the Hardy space $H^1(\mathbb{R}^n)$. Let $P(x)$ be a polynomial, $\nabla P(0) = 0$, and $K(x)$ be a Calderón–Zygmund kernel. Define T :

$$(1.1) \quad Tf(x) = \text{p.v.} \int_{\mathbb{R}^n} e^{iP(x-y)} K(x-y) f(y) dy.$$

It was shown in [6] that T is a bounded operator on H^1 , and the norm of T depends only on the degree of P , not its coefficients.

The purpose of the present paper is to study operators of the form (1.1), but with general smooth phase functions. More specifically, we will be dealing with the following (localized) oscillatory singular integral operators:

$$(1.2) \quad T_\lambda : f \rightarrow \text{p.v.} \int_{\mathbb{R}^n} e^{i\lambda\Phi(x-y)} K(x-y) \varphi(x-y) f(y) dy,$$

where $\lambda \in \mathbb{R}^1$, $\Phi \in C^\infty$, $\varphi \in C_0^\infty$, and K is a Calderón–Zygmund kernel. Our main interest is in studying the uniform boundedness of T_λ on H^1 (it is easily seen that each T_λ is bounded on H^1). The result in [6] implies that the T_λ operators are uniformly bounded on H^1 provided that $\Phi(x)$ is a real-valued polynomial with $\nabla\Phi(0) = 0$.