

# *Spectral Theory of Operators of Hankel Type, II*

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ABSTRACT. For smooth real  $K(x)$ , the self-adjoint singular integral operator

$$Kf(x) = \int_0^\infty K(x+y)f(y) dy$$

has no singular continuous part, non-zero point eigenvalues of finite multiplicity accumulating only at zero, and an absolutely continuous part determined by the limits of  $xK(x)$  at zero and infinity.

**Introduction.** This paper discusses the spectral theory of self-adjoint integral operators on  $L_2(0, \infty)$  with sum kernels

$$Kf(x) = \int_0^\infty K(x+y)f(y) dy$$

[5, Chapter 2]. With sufficient smoothness on  $K(x)$ , it will be shown that

- (i) the non-zero eigenvalues of  $H$  are of finite multiplicity and can accumulate only at zero;
- (ii)  $H$  has no singular continuous spectrum, and
- (iii) the absolutely continuous spectrum of  $H$  is the direct sum of the multiplications by  $\lambda$  on  $L_2[0, \pi a]$  and  $L_2[0, \pi b]$  where  $a$  and  $b$  are the limits at zero and infinity of  $k(x) = xK(x)$ .

If

$$K(x) = \int_0^\infty e^{-xs} |\varphi(x)|^2 ds,$$

then (under technical conditions on  $\varphi$ ),  $H$  is unitarily equivalent to the positive definite operator

$$(I.1) \quad H(\varphi, \varphi) = \int_0^\infty \frac{\varphi(x)\bar{\varphi}(y)}{x+y} f(y) dy$$