

Analytic Disks and Pluripolar Sets

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0. Introduction. Let \mathcal{A} be the disk algebra of holomorphic functions in the unit disk U which are continuous in the closed disk $\bar{U} = U \cup S$, where $S = \{z : |z| = 1\}$ is the unit circle. For a holomorphic function f on U , we let

$$A_f = \{(z, f(z)) : z \in U\}$$

be the graph of f in \mathbb{C}^2 lying over U and, if $f \in \mathcal{A}$, we let

$$\Gamma_f = \{(z, f(z)) : z \in S\}$$

be the graph of f over S .

The set A_f is an example of a *pluripolar set*: a subset E of \mathbb{C}^2 is pluripolar if there is a plurisubharmonic function u in \mathbb{C}^2 with $\{(z, w) \in \mathbb{C}^2 : u(z, w) = -\infty\}$ containing E (a function is plurisubharmonic if it is uppersemicontinuous and its restriction to complex lines in \mathbb{C}^2 is subharmonic). Pluripolarity can also be defined locally; it is a result of Josefson [J] that the local definition coincides with the global one. Subsets of analytic varieties are standard examples of pluripolar sets in \mathbb{C}^n . For instance, if $g : \mathbb{C} \rightarrow \mathbb{C}$ is entire, then for any subset N of \mathbb{C} the graph of g over N ,

$$(0.1) \quad E = \{(z, g(z)) : z \in N\},$$

is pluripolar. If N is not polar as a subset of \mathbb{C} , then any plurisubharmonic function u such that $u = -\infty$ on E is necessarily equal to $-\infty$ on the variety

$$(0.2) \quad F = \{(z, g(z)) : z \in \mathbb{C}\}.$$

Thus we see the phenomenon of propagation of pluripolar sets; the set F of (0.2) is the exact $-\infty$ locus of a plurisubharmonic function (for instance, $\log|w - g(z)|$). We call such sets *complete* pluripolar sets.

The main results of our paper show that, unlike the case of the graphs of entire functions, if $f \in \mathcal{A}$, the sets A_f and $A_f \cup \Gamma_f$ can exhibit much different behaviour with respect to these notions of pluripolarity and complete pluripolarity. We begin by analyzing in more detail the graphs of certain gap series f (which may or may not lie in \mathcal{A}) as first studied by Sadullaev [S1]. These graphs A_f are always complete pluripolar (see Proposition 2.4), and if we require, in addition,