

An Absolute Gradient Bound for Nonparametric Surfaces of Constant Mean Curvature

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ABSTRACT. As is well known, no manifold of constant mean curvature 1 can exist in a domain strictly containing the unit ball. We shall consider the problem of estimating absolute a priori bounds for a manifold of constant mean curvature 1 in a ball of radius less than 1 without imposing boundary conditions or bounds of any sort in the two-dimensional case.

In a paper by Finn and Giusti, the existence of an absolute gradient bound near the center has been verified for nonparametric surfaces of constant mean curvature 1 in a disk of radius larger than 0.56540... and less than 1. The purpose of this paper is to determine explicitly the largest subregion inside such a disk in which such an absolute gradient bound holds. The method used in this work is to reduce the problem to a two-dimensional variational problem with the help of the notion of generalized solutions introduced by Miranda. A capillary surface is used as a comparison extremal surface.

From the outset, I wish to thank my dissertation advisor Robert Finn. He suggested the problem considered in this paper, offered crucial advice for the proofs and the formulations of the results, and has been scrupulous in rectifying my manuscript. Moreover, the main idea of Sections 2 and 3 in this paper is intrinsically related to parts of Section 7.11 in his book [4] (or Chapters II, III of [6]).

In this paper, we shall consider the problem of estimating absolute a priori gradient bounds for the solutions of

$$\operatorname{div} Tu = 2,$$

where $Tu = \nabla u / (\sqrt{1 + |\nabla u|^2})$ is a 2-dimensional disk B_R , $R < 1$, without imposing boundary conditions or bounds of any sort on the solutions. The first section is devoted to some introductory material, from which the motivation for this problem will become clear.