

Poincaré Inequalities for a Class of Non- A_p Weights

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ABSTRACT. We derive Poincaré inequalities for a pair L_u^q, L_v^p of weighted norm spaces, $1 < p \leq q < \infty$, in case u and v are related by a natural condition of dimensional balance and v is a finite product of power weights times an A_p function.

1. Introduction. Our goal is to enlarge the class of pairs of weight functions $u(x)$, $v(x)$ on n -dimensional Euclidean space \mathbb{R}^n for which the following Poincaré inequality is known to be valid:

$$(1.1) \quad \left(\frac{1}{u(B)} \int_B |f(x) - f_B|^q u(x) \, dx \right)^{1/q} \leq c|B|^{1/n} \left(\frac{1}{v(B)} \int_B |\nabla f(x)|^p v(x) \, dx \right)^{1/p}.$$

Here, $1 < p \leq q < \infty$, B is a ball in \mathbb{R}^n with volume $|B|$, u and v are nonnegative locally integrable functions on \mathbb{R}^n , $u(B) = \int_B u(x) \, dx$, $v(B) = \int_B v(x) \, dx$, f_B is a constant independent of x , ∇f is the gradient of f , and c is a constant independent of f and B . The inequality is required to hold for all balls B and all $f \in \text{Lip}(B)$, the class of Lipschitz functions on B .

We say that u satisfies the doubling condition, or that u is a doubling weight, if there is a constant c so that $u(2B) \leq cu(B)$ for all balls B , where $2B$ denotes the ball with the same center as B and twice the radius. We shall assume that both u and v are doubling weights. In this case, a necessary condition for (1.1) to hold for all balls is

$$(1.2) \quad \left(\frac{|B'|}{|B|} \right)^{1/n} \left(\frac{u(B')}{u(B)} \right)^{1/q} \leq c \left(\frac{v(B')}{v(B)} \right)^{1/p}, \quad B' \subset B,$$

where B' and B are balls and c is a constant independent of the balls (see [CW1]).

A variety of conditions are known to be sufficient for (1.1). Among these, the ones of greatest interest to us here are contained in [CW1], [CW2], [SaW]