

# *Stability and Bifurcation of a Surface of Constant Mean Curvature in a Wedge*

THOMAS I. VOGEL

ABSTRACT. The problem considered is that of a surface of constant mean curvature making a constant contact angle with two planes forming a wedge. The physical problem motivating this is to determine the behavior of liquid wetting the edge of a wedge. The only energies involved are surface tension and the energy that the liquid gains by wetting the planes. In particular, the problem is static and we do not include gravity. It is shown that if the contact angle  $\gamma$  and the opening angle of the wedge  $2\alpha$  satisfy  $\alpha + \gamma \leq \pi/2$  then a cylindrical surface is stable and is unique among periodic stationary surfaces. If  $\alpha + \gamma > \pi/2$ , then the cylindrical surfaces are no longer stable, uniqueness fails, and in fact the existence of a family of non-cylindrical stationary surfaces is demonstrated.

**1. Introduction.** Much of the study of capillary phenomena has been motivated by a desire to understand the behavior of liquid in microgravity (e.g., in spacecraft), where surface forces will predominate. Although the behavior of a cylinder of liquid subject to surface tension is a well-studied problem (this is the Rayleigh instability), the behavior of a column of liquid in a solid wedge has received little attention. This case is important, however, for the design of tanks to hold liquid in spacecraft. Indeed, if the contact angle is such that the liquid in a tank gains energetically by wetting the corners of the tank, it will be difficult to empty it efficiently. A stationary solution is given by a section of a circular cylinder. Natural questions to ask are for what values of the contact angle and the opening angle of the wedge are these cylindrical surfaces stable, are there other stationary surfaces, and if so are they stable.

Langbein ([7]) treated a linearization of the problem, and derived a stability criterion for the cylinder by observing a bifurcation in the linear problem. However, bifurcation in a linearized problem does not necessarily imply that the original non-linear problem bifurcates (see [2] for examples of this), and it is not