

The Dirichlet Problem for Elliptic Systems in Piecewise C^1 Plane Domains

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ABSTRACT. We study the Dirichlet problem for a strongly elliptic system $Lu = 0$ with boundary data in $L^p(\partial\Omega)$, $1 < p < +\infty$, in a piecewise C^1 plane domain Ω . We show that it is always possible to represent the solution in the form of a double layer potential for which the boundary integral operator is always a Fredholm operator of index 0 for all p , $2 \leq p < +\infty$.

Introduction. Boundary value problems associated with the systems of elastostatics and hydrostatics have been recently studied by Dahlberg, Fabes, Kenig and Verchota, [1], [2], [3], [8], on Lipschitz domains in \mathbb{R}^n . They use the layer potential method reducing the problem to that of an integral equation on the boundary. The invertibility of the boundary integral operator yields a solution in $L^2(\Omega)$. Well known arguments extend their results for $2 - \epsilon \leq p \leq 2 + \epsilon$, where ϵ is a constant depending on the Lipschitz domain Ω . In the special case of a curvilinear polygonal domain Ω in the plane, J. Lewis, [11], studies the same type of boundary value problems in $L^p(\Omega)$, $1 < p < +\infty$. He develops the theory of double layer potentials which are interpreted as systems of pseudodifferential operators of Mellin type. A symbolic calculus of Mellin operators was developed by J. Lewis and C. Parenti, [10], and J. Elschner, [2]. J. Lewis, [11], shows, in particular, that the boundary system of integral equations is a Fredholm operator of index 0 on $L^p(\partial\Omega)$ for all p , $2 \leq p < +\infty$, and finds its singularities.

In this paper we study the Dirichlet problem for a general second order elliptic system on a bounded plane domain Ω whose boundary $\partial\Omega$ is a simple, closed, continuous piecewise C^1 curve. We show that it is always possible to represent the solution in the form of a double layer potential for which the boundary integral operator is a Fredholm operator of index 0 for all p , $2 \leq p < +\infty$. The Fredholm and index properties are obtained thanks to the results of Lewis, [12], who develops a symbolic calculus for layer potentials on C^1 curves. We deal with a system of partial differential equations of the form