

Generalized Flow of Sets by Mean Curvature on a Manifold

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ABSTRACT. The level-set flow of Evans-Spruck and Chen-Giga-Goto is generalized to a Riemannian manifold, using recent techniques of Crandall-Ishii for viscosity solutions. Generally speaking, the motion is not unique for noncompact closed sets, but the definition can be modified to make the motion unique. We give examples to show: (1) a smooth set can develop an interior that originates from infinity (2) in the case of a Grayson neckpinch, the evolving function $u(x, t)$ need not remain C^2 .

Recently, Chen, Giga, and Goto [CGG] and Evans and Spruck [ES1, ES2, ES3, ES4] have defined a generalized mean curvature flow of sets in \mathbf{R}^n . In the present paper, we extend this construction to a Riemannian manifold M . We also discuss the generalization to noncompact closed sets, and exhibit some of its pathologies. Finally, we record an observation about the smoothness of the function used in the construction. Recent work of Crandall and Ishii [CI] simplifies the technicalities of “viscosity” solutions needed in the proofs.

The results of this paper and some additional material are reported in [I1].

Brakke began the study of the generalized mean curvature flow in [B] using methods of geometric measure theory. In the eighties, Gage [G], and Gage and Hamilton [GH] established the regularity of convex curves evolving in the plane. Grayson extended the smoothness to general embedded curves in [Gr1, Gr3], solving a problem posed by Gauss. Angenent has recently given regularity theory for immersed curves in [An2, An3, An4]. Huisken has studied convex bodies of higher dimension in [H1] and [H2], and singularity formation in [H3]. These approaches all employ classical differential geometry and the maximum principle. An approach using the singular limit of a modified heat equation appears in de Mottini and Schatzman [MS1, MS2], Bronsard and Kohn [BK], De Giorgi [D], Evans, Soner, and Souganidis [ESS], Ilmanen [I3] and elsewhere. Soner [So] has given a viscosity definition directly for the moving sets themselves.