

On the Automorphisms of Special Domains in \mathbb{C}^n

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ABSTRACT. Let K be a compact Lie group which acts linearly on \mathbb{C}^n . Assume that every K -invariant holomorphic function on \mathbb{C}^n is constant. For every K -invariant bounded domain $\Omega \subset \mathbb{C}^n$ which contains the origin we prove that the holomorphic automorphisms of Ω are holomorphically extendable over the boundary of Ω . Furthermore we give some results concerning the structure of $\text{Aut}_{\mathcal{O}}(\Omega)$

A domain Ω in \mathbb{C}^n is called a (p_1, \dots, p_n) -domain if it is stable under the transformations

$$(z_1, \dots, z_n) \longrightarrow (t^{p_1} z_1, \dots, t^{p_n} z_n)$$

where $t \in S^1 = \{s \in \mathbb{C}; |s| = 1\}$ and p_1, \dots, p_n are integers. If Ω is a bounded (p_1, \dots, p_n) -domain containing the origin and $p_j > 0$ for all j , then W. Kaup ([K]) has proved the following:

- (i) *Every biholomorphism of Ω extends holomorphically to an open neighbourhood of the topological closure of Ω .*
- (ii) *The group $G = \text{Aut}(\Omega)$ of holomorphic automorphisms of Ω has only finitely many connected components.*
- (iii) *The orbit $G \cdot 0 = \{g(0); g \in G\}$ is a connected, closed complex submanifold of Ω .*

Our goal here is to extend this result to a more general class of domains.

Let K be a compact Lie group and let $\rho : K \rightarrow GL(\mathbb{C}^n)$ be a continuous representation such that the algebra

$$\mathcal{O}(\mathbb{C}^n)^K = \{f \in \mathcal{O}(\mathbb{C}^n); f \circ \rho(k) = f \text{ for all } k \in K\}$$

of K -invariant holomorphic functions consists only of the constant functions. Then for any K -invariant domain Ω in \mathbb{C}^n which contains the origin it follows that $\mathcal{O}(\Omega)^K = \mathbb{C}$ (see Section 1). Under the assumption that the domain Ω is bounded the results of this note are easy to formulate. In this case the statements (i), (ii) and (iii) are true, and furthermore: