

On Removable Singularities of Complex Analytic Sets

A. B. SUKHOV

ABSTRACT. This article considers boundary properties of complex analytic sets. The possibility of the continuation of complex analytic sets across a real submanifold in \mathbf{C}^n whose Levi form is (in some sense) nondegenerate is shown.

0. Introduction. In this paper we study the well-known problem of the analytic continuation of complex analytic sets. There are two main directions in this problem: the local analytic continuation and the removal of singularities. Let Ω be a domain in \mathbf{C}^N and M is a closed subset in Ω . Suppose that A is a complex purely q -dimensional analytic subset in $\Omega \setminus M$ (as usual, we always suppose that A is closed in $\Omega \setminus M$). Then M is called a removable singularity for A if a closure \bar{A} of A (relatively Ω) is an analytic subset in Ω . We say that a purely q -dimensional analytic subset A of a domain $D \subset \mathbf{C}^N$ analytically continues to the neighborhood U of a point $p \in \bar{A} \cap \partial D$ if there exists a complex purely q -dimensional subset $\tilde{A} \subset U$ such that $(A \cap U) \subset \tilde{A}$.

One of the most famous results about removable singularities is the Remmert-Stein theorem [Re, St], which guarantees the removal of M under the condition that M be a complex analytic set and $\dim M < \dim A$. Essentially, this result means that if a (real) metrical dimension of M is sufficiently small in comparison with a real dimension of A , then M is removable for A (see [Sh1]). The notion of complex manifold is a partial case of the notion of CR-manifold (Cauchy-Riemann manifold). Recall that a real submanifold M in \mathbf{C}^N is called a CR-manifold if the dimension of its holomorphic tangent space does not depend on the choice of a point of M (it is called a CR-dimension of M and denotes $\text{CR dim } M$). In [Be], [Sh2], [Fu], [Ch1] a significant fact was discovered, namely, that a CR-manifold M whose real dimension is arbitrarily more than the real dimension of A can be removable for A if the CR-dimension of M is sufficiently less than $\dim A$. In the present paper we use the Levi form of a CR-manifold instead of a CR-dimension. By comparison to the CR-dimension, the Levi form is essentially a more delicate characteristic of the biholomorphic