

Composition Operators on Large Weighted Bergman Spaces

THOMAS L. KRIETE III & BARBARA D. MACCLUER

ABSTRACT. Composition operators, considered as linear transformations acting on very general weighted Bergman spaces on the unit disk, are studied. Criteria for boundedness, compactness, and membership in the Schatten classes are presented.

1. Introduction. Let φ be an analytic self-map of the open unit disk $\mathbb{D} = \{z : |z| < 1\}$ in the complex plane. In this paper we study the associated composition operator C_φ , defined by $C_\varphi(f) = f \circ \varphi$, considered as acting on very general weighted Bergman spaces in \mathbb{D} . Much is known about the structure of C_φ as an operator on the Hardy space H^2 and on those weighted Bergman spaces associated with the “standard weights.” We will show here that the behavior of C_φ changes radically when one moves beyond the standard weights into the realm of “fast weights.” We investigate how the weight function and φ determine whether C_φ is bounded, compact, or in some Schatten ideal \mathcal{S}_p . The most fundamental property, boundedness, is simply equivalent, by the closed graph theorem, to the requirement that the space of functions in question be closed under composition with φ .

To be more precise, let dA denote Lebesgue area measure on \mathbb{D} and suppose G is a positive continuous function on $(0, 1)$ such that $G(|z|)dA(z)$ defines a finite measure on \mathbb{D} ; that is, $\int_0^1 G(r)r dr < \infty$. The weighted Bergman space (with weight G) will be denoted A_G^2 and is defined to be the collection of all functions analytic on \mathbb{D} for which

$$(1.1) \quad \|f\|^2 \equiv \int_{\mathbb{D}} |f(z)|^2 G(|z|) dA(z) < \infty.$$

One checks that if $f = \sum_0^\infty a_n z^n$, then $\|f\|^2 = 2\pi \sum_{n=0}^\infty p_n |a_n|^2$, where $p_n = \int_0^1 r^{2n+1} G(r) dr$. Moreover, A_G^2 is a Hilbert space with norm $\|\cdot\|$ defined by (1.1). We sometimes write $\|\cdot\|_G$ for $\|\cdot\|$. The weights $G(r) = (1-r)^\alpha$, $\alpha > -1$, are one version of the standard weights; other versions, which give equivalent norms, are described in Section 2. The moments p_n corresponding to the standard weights