

# *Some $L^p$ Estimate for the Exterior Stokes Flow and An Application to the Non-Stationary Navier-Stokes Equations*

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ABSTRACT. In  $n(\geq 3)$ -dimensional exterior domains  $\Omega$ , it is known that if  $1 < p < n/2$ , the estimate  $\|D^2u\|_p \leq C\|A_p u\|_p$  holds for all  $u \in D(A_p)$ , where  $A_p$  is the Stokes operator in  $L^p_r(\Omega)$ . If  $n/2 \leq p$ , a counter example defeats validity of this estimate. If we set an additional term on the right hand side, the second derivatives in  $L^p(\Omega)$ , however, can be dominated as follows.

$$\|D^2u\|_p \leq C(\|A_p u\|_p + \|\nabla u\|_r), \quad u \in D(A_p),$$

where  $p \leq r < \infty$ . Our result also covers the case  $n = 2$ .

As an application, we obtain an asymptotic behavior of second derivatives in  $L^p(\Omega)$  for weak and strong solutions to the non-stationary Navier-Stokes equations.

**0. Introduction.** Let  $\Omega$  be an exterior domain in  $\mathbb{R}^n$  ( $n \geq 2$ ), i.e., a domain having a compact complement  $\mathbb{R}^n \setminus \Omega$ , and assume that the boundary  $\partial\Omega$  is of class  $C^{2+\mu}$  with  $0 < \mu < 1$ . Consider the boundary-value problem for the Stokes equations in  $\Omega$ :

$$(S-S) \quad \begin{cases} -\Delta u + \nabla p = f, & x \in \Omega, \\ \operatorname{div} u = 0, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases}$$

where  $u(x) = (u_1(x), u_2(x), \dots, u_n(x))$  and  $p(x)$  denote the unknown velocity vector and pressure at a point  $x \in \Omega$ , respectively, while  $f(x) = (f_1(x), f_2(x), \dots, f_n(x))$  is a given external force. In this paper, we are mainly interested in an a-priori estimate in  $L^p$  for a pair of the solution  $(u, p)$  of (S-S). There is a large literature on  $L^p$ -theory for the Stokes equations (cf. [22], [31]). When  $\Omega$  is a