

Asymptotic Behavior of Solutions to the Three-Dimensional Navier-Stokes Equations

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1. Introduction. We consider the asymptotic behavior of solutions to the Navier-Stokes equations in three space dimensions

$$(N-S) \quad \begin{aligned} u_t + u \cdot \nabla u + \nabla p &= \Delta u, \\ \operatorname{div} u &= 0. \end{aligned}$$

This paper is a continuation of work started in [9] where we established lower bounds to the solutions in two spatial dimensions and non-uniform lower bounds for solutions in three spatial dimensions to the Navier-Stokes equations. The purpose of this paper is to show that solutions to the three-dimensional Navier-Stokes equations also have uniform lower bounds of rates of decay.

More precisely, it is shown that if $u(x, t)$ is a Leray-Hopf solution to the three-dimensional Navier-Stokes equations in the sense of Caffarelli, Kohn and Nirenberg [1] with zero average initial data outside a class of functions of radially equidistributed energy, then there exist constants C_0 and C_1 depending only on norms of the initial data such that

$$C_0(t+1)^{-5/2} \leq \|u(\cdot, t)\|_{L^2}^2 \leq C_1(t+1)^{-5/2}.$$

We recall that in [7, 8] we showed that for non-zero average initial data there exist constants C_2, C_3 depending only on the L^2 and L^1 norms of the initial data such that

$$C_2(t+1)^{-3/2} \leq \|u(\cdot, t)\|_{L^2}^2 \leq C_3(t+1)^{-3/2}.$$

For initial data which have radially equidistributed energy, an extension of an example suggested by A. Majda for the two-dimensional Navier-Stokes shows that solutions can be constructed which decay exponentially, showing that the condition of radially equidistributed energy on the data is necessary.

In Section 2 standard notation is recalled and some theorems which were proved in [9] are stated. In Section 3 it is shown that the solutions to the three-dimensional Navier-Stokes equations admit an algebraic uniform lower bound rate of decay.