

An Upper Bound for the Winding Number for Solutions of the Ginzburg–Landau Equation

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ABSTRACT. In the case of the 1D complex Ginzburg-Landau equation, it is proved that there exists an upper bound for the winding number of any function on the attractor, around any point in the complex plane for which the definition makes sense. It is proved that this upper bound depends only on the bifurcation parameters of the equation. As auxiliary results, an estimate for the space analyticity radius is provided, and the strong squeezing property is extended to higher order derivatives.

1. Introduction. We will study the Ginzburg-Landau equation (GLE) on \mathbf{R} , which reads as

$$(1.1) \quad \frac{\partial u}{\partial t} - (1 + i\nu) \frac{\partial^2 u}{\partial x^2} + (1 + i\mu)|u|^2 u - au = 0$$

$$(1.2) \quad u(x, 0) = u_0(x), \quad x \in \mathbf{R}$$

$$(1.3) \quad u(x + 1, t) = u(x, t), \quad x \in \mathbf{R}, t \geq 0$$

where $a > 0$, ν , μ are given real numbers, and u_0 belongs to the Hilbert space H formed by complex valued functions with period 1 which are in $L^2_{\text{loc}}(\mathbf{R})$.

This equation appears in the study of the amplitude of unstable wave-like solutions of many equations in hydrodynamics (see [10] for references as well as for the basic properties of this equation).

Among the natural parameters describing the complexity of a solution of the GLE is its winding number as a periodic function in x , around any point outside its range. More precisely, let $f: \mathbf{R} \rightarrow \mathbf{C}$ be a differentiable periodic function with period 1, and denote $\{f\} = \{f(x) : x \in [0, 1]\} \subseteq \mathbf{C}$; then the winding number of f around a point $p \notin \{f\}$ is defined as

$$\text{ind}_p(f) = \frac{1}{2\pi i} \int_0^1 \frac{f'(x)}{f(x) - p} dx.$$