

A Geometrical Characterization of Intrinsic Ultracontractivity for Planar Domains with Boundaries Given by the Graphs of Functions

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ABSTRACT. Let $D = \{(x, y) : 0 < x < 1, f(x) < y < 1\}$, where f is a nonpositive uppersemicontinuous function. Let A_r be all those connected subsets B of D which lie below the line $y = r$ and such that $B \cap \{y = s\}$ is connected for each s . Then the semigroup associated with the heat kernel for D is intrinsically ultracontractive if and only if

$$(*) \quad \lim_{r \rightarrow -\infty} \sup_{B \in A_r} \text{area}(B) = 0.$$

A simply connected union of a finite number of sets of the form D , in perhaps different orthonormal coordinate systems, has an intrinsically ultracontractive semigroup if and only if each of the sets satisfies (*).

0. Introduction. Let D be a domain in \mathbb{R}^d , $d \geq 2$, and let $P_t^D(z, w)$, $t > 0$, $z, w \in D$, be the heat kernel for one half the Dirichlet Laplacian in D . We assume that D has a positive eigenfunction φ in $L^2(dz)$ with eigenvalue λ , an assumption which holds for all domains considered in this paper. Since

$$P_t^D(z, w) \leq \frac{1}{(2\pi t)^{d/2}} e^{-(|z-w|^2/2t)},$$

the Markovian semigroup associated with $P_t^D(z, w)$ is ultracontractive. That is, it maps $L^2(D, dz)$ into $L^\infty(D, dz)$ for all $t > 0$. Following Davies and Simon [10] we shall say that D is *intrinsically ultracontractive*, which henceforth we write as IU, if the new Markovian semigroup in $L^2(\varphi^2 dz)$ with kernel

$$\tilde{P}_t(z, w) = \frac{e^{\lambda t} P_t^D(z, w)}{\varphi(z)\varphi(w)},$$