

# *Extension Theorems on Weighted Sobolev Spaces*

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ABSTRACT. Let  $w_i \in A_{p_i}$ ,  $1 \leq p_i < \infty$  for  $i = 1, 2, \dots, N$ . For any unbounded  $(\varepsilon, \infty)$  domain  $\mathcal{D}$ , by modifying a technique of P. Jones (cf. [11]), we show that there exists an extension operator  $\Lambda$  on  $\mathcal{D}$  such that

$$\|\nabla^{k_i} \Lambda f\|_{L_{w_i}^{p_i}(\mathbb{R}^n)} \leq C_i \|\nabla^{k_i} f\|_{L_{w_i}^{p_i}(\mathcal{D})}$$

for all  $i$  where  $C_i$  depends only on  $\varepsilon$ ,  $w_i$ ,  $k_i$ ,  $n$  and  $\max_i k_i$ . Moreover, when  $\mathcal{D}$  is a bounded  $(\varepsilon, \infty)$  domain, a similar but weaker result holds. We also extend P. Jones' result on  $(\varepsilon, \delta)$  domains to  $A_p$ -weighted Sobolev spaces. Finally, many applications such as Sobolev interpolation inequalities and Nirenberg-type inequalities on  $(\varepsilon, \infty)$  domains are given.

**Introduction.** Let  $\mathcal{D}$  be an open set in  $\mathbb{R}^n$ . If  $\alpha$  is a multi-index,  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n) \in \mathbb{Z}_+^n$ , we will denote  $\sum_{j=1}^n \alpha_j$  by  $|\alpha|$  and  $D^\alpha = (\partial/\partial x_1)^{\alpha_1} \dots (\partial/\partial x_n)^{\alpha_n}$ . By  $\alpha \geq \beta$ , we mean  $\alpha_j \geq \beta_j$  for all  $1 \leq j \leq n$ . Moreover, we write  $\alpha > \beta$  if  $\alpha \geq \beta$  and  $\alpha \neq \beta$ . We denote by  $\nabla$  the vector  $(\partial/\partial x_1, \partial/\partial x_2, \dots, \partial/\partial x_n)$  and by  $\nabla^m$  the vector of all possible  $m^{\text{th}}$ -order derivatives for  $m \in \mathbb{N}$ . A locally integrable function  $f$  on  $\mathcal{D}$  has a weak derivative of order  $\alpha$  if there is a locally integrable function (denoted by  $D^\alpha f$ ) such that

$$\int_{\mathcal{D}} f(D^\alpha \varphi) dx = (-1)^{|\alpha|} \int_{\mathcal{D}} (D^\alpha f) \varphi dx$$

for all  $C^\infty$  functions  $\varphi$  with compact support in  $\mathcal{D}$  (we will write  $\varphi \in C_0^\infty(\mathcal{D})$ ).

If  $1 < p < \infty$ ,  $p'$  is always equal to  $p/(p-1)$  and  $p' = \infty$  when  $p = 1$ .  $Q$  will always be a cube and  $\ell(Q)$  will be its edglength. We say that two cubes touch if a face of one cube is contained in a face of the other. By a weight  $w$ , we mean a non-negative locally integrable function on  $\mathbb{R}^n$ . By abusing notation, we will also write  $w$  for the measure induced by  $w$ . Sometimes we write  $dw$  to denote  $w dx$ . We are only interested in the case when  $w$  and Lebesgue measure are mutually absolutely continuous. Moreover, we always assume  $w$  is doubling,