

An Inverse Problem Originating from Magnetohydrodynamics II: The Case of the Grad-Shafranov Equation

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ABSTRACT. The so-called Grad-Shafranov equation is a semi-linear elliptic equation which is commonly used to model the plasma equilibrium in a Tokamak. We study an inverse problem associated with this equation. We show that knowledge of the normal derivative of the poloidal magnetic flux on the plasma boundary uniquely determines the functional form of the source terms within the class of analytic functions, provided the boundary has a (certain type of) corner. This result may in some ways be seen as an extension of a previously established result for the equation $\Delta u = -f(u) \leq 0$, cf. [2].

Introduction. According to a commonly used model for the magnetohydrodynamic (MHD) equilibrium in a Tokamak, the axisymmetric equilibrium state is completely determined by two functions: the toroidal field function $f(\psi)$ and the pressure profile $p(\psi)$, together with appropriate boundary conditions. The associated poloidal magnetic flux, ψ , satisfies the so called Grad-Shafranov equation, which in natural cylindrical coordinates (r, φ, z) takes the rotationally symmetric form

$$(0.1) \quad \frac{\partial}{\partial r} \left(\frac{1}{\mu_0 r} \frac{\partial \psi}{\partial r} \right) + \frac{\partial}{\partial z} \left(\frac{1}{\mu_0 r} \frac{\partial \psi}{\partial z} \right) = -r \frac{dp}{d\psi} - \frac{1}{\mu_0 r} f \frac{df}{d\psi} \quad (\leq 0).$$

μ_0 denotes the constant magnetic permeability, which in the following we shall always take to be equal to 1. The equation (0.1) is satisfied in a domain Ω , the cross section of the plasma region, independent of angle. For more details about the derivation of this equation from general principles we refer the reader to [12]. Since the plasma is in equilibrium the flux, ψ , is constant on the boundary; for simplicity we take

$$(0.2) \quad \psi = 0 \quad \text{on } \partial\Omega.$$