

# *Spherically Symmetric Solutions of the Navier-Stokes Equations for Compressible, Isothermal Flow with Large, Discontinuous Initial Data*

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**ABSTRACT.** We construct global weak solutions of the Navier-Stokes equations for viscous, compressible, isothermal flow, when the Cauchy data is large, discontinuous, and spherically symmetric. A great deal of technical and qualitative information is obtained about these solutions. For example, we show that solutions may develop vacuum regions about the origin, and we prove that the boundaries of these regions are Hölder continuous in space-time. We also show that discontinuities in the velocity become smoothed out in positive time, and that discontinuities in the density and pressure persist for all time, convecting along particle trajectories, and decaying at a rate inversely proportional to the viscosity coefficient.

**1. Introduction.** We construct global solutions of the Navier-Stokes equations

$$(1.1) \quad \begin{cases} \rho_t + \operatorname{div}(\rho \mathbf{U}) = 0, \\ (\rho \mathbf{U}^j)_t + \operatorname{div}(\rho \mathbf{U}^j \mathbf{U}) + P(\rho)_{x_j} = \varepsilon_1 \Delta \mathbf{U}^j + \varepsilon_2 \operatorname{div} \mathbf{U}_{x_j}, \end{cases}$$

for viscous, compressible, isothermal flow, with Cauchy data

$$(1.2) \quad \rho(\cdot, 0) = \rho_0, \quad \mathbf{U}(\cdot, 0) = \mathbf{U}_0,$$

which is large, discontinuous, and radially symmetric. Here  $\rho(x, t)$ ,  $\mathbf{U}(x, t)$ , and  $P(\rho(x, t))$  represent the density, velocity, and pressure in the fluid,  $\varepsilon_1$  and  $\varepsilon_2$  are positive viscosity constants,  $t \geq 0$  is time, and  $x \in \mathbb{R}^n$ ,  $n \geq 2$ , is the spatial coordinate.  $\mathbf{U}^j$  is the  $j^{\text{th}}$  component of the  $n$ -vector  $\mathbf{U}$ , and  $\operatorname{div}$  and  $\Delta$  are the usual spatial divergence and Laplace operators. We construct solutions of