

On the Mapping Problem for Quadric Cauchy-Riemann Manifolds

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ABSTRACT. This note establishes a sufficient condition for the rationality of a holomorphic mapping between quadric Cauchy-Riemann manifolds.

0. Introduction. One of the most significant phenomena of the mapping problem in several complex variables is the following effect: every local biholomorphism of the unit sphere of \mathbb{C}^n , $n > 1$, extends to a complex rational mapping on all \mathbb{C}^n . For $n = 2$, this result is due to H. Poincaré [Po]; the general case was considered by N. Tanaka [Ta], D. Pelles [Pe] and H. Alexander [A1]. J. Cima and T. J. Suffridge [CS], F. Forstnerič [For1] and V. Rabotin [Ra] obtained similar results for mappings of unit spheres of complex linear spaces of different dimensions. We remark also that the unit sphere is a strongly pseudoconvex quadric of codimension 1. A. Tumanov and G. Henkin [TuHe] and F. Forstnerič [For2] established sufficient conditions for the rational extension in the mapping problem for strongly pseudoconvex quadric Cauchy-Riemann manifolds of arbitrary codimensions. In the present note we show that the strong pseudoconvexity can be replaced by some weaker condition in these results. Our main tool is the reflection principle of H. Lewy [Le], S. Pinchuk [Pi1] and S. Webster [We]. We also use a special argument (“the elimination method”) due to S. Pinchuk [Pi3].

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1. Result. Consider in \mathbb{C}^n coordinates $z = ('z, ''z)$, where $'z = (z_1, \dots, z_k) \in \mathbb{C}^k$, $''z = (z_{k+1}, \dots, z_n) \in \mathbb{C}^d$, $k + d = n$, $k, d > 0$. Then a real generic manifold M in \mathbb{C}^n , defined by equations

$$(1.1) \quad ''z_j + ''\bar{z}_j = \langle L_j('z), 'z \rangle, \quad j = 1, \dots, d,$$