

Orbifolds of Maximal Diameter

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ABSTRACT. In this paper the Maximal Diameter Theorem of Riemannian geometry is proven for Riemannian orbifolds. In particular, it is shown that a complete Riemannian orbifold with Ricci curvature bounded below by $(n - 1)$ and diameter $= \pi$, must have constant sectional curvature 1, and must be a quotient of the sphere (S^n, can) of constant sectional curvature 1 by a subgroup of the orthogonal group $\mathcal{O}(n + 1)$ acting discontinuously and isometrically on S^n . It is also shown that the singular locus of the orbifold forms a geometric barrier to the length minimization property of geodesics. We also extend the Bishop relative volume comparison theorem to Riemannian orbifolds.

Introduction. In this paper we wish to examine a generalization of the maximal diameter theorem of Cheng to Riemannian orbifolds. Roughly speaking, a Riemannian orbifold is a metric space locally modelled on quotients of Riemannian manifolds by finite groups of isometries. The term *orbifold* was coined by W. Thurston sometime around the year 1976–77. The term is meant to suggest the orbit space of a group action on a manifold. A similar concept was introduced by I. Satake in 1956, where he used the term *V-manifold* (See [S1]). The “V” was meant to suggest a cone-like singularity. Since then, orbifold has become the preferred terminology. Riemannian orbifolds have recently come up in the study of convergence of Riemannian manifolds. For instance, Anderson [A] has shown that metric spaces in the Gromov–Hausdorff closure of the set of compact Riemannian manifolds with bounded Ricci curvature, lower volume bound, uniform upper diameter bound, and upper bound on the $L^{n/2}$ -norm of the Ricci tensor have the structure of an orbifold with only a finite number of singular points. Also, Fukaya [F] has shown that if a sequence of manifolds $\{M_i\}$ in the class of compact n -dimensional Riemannian manifolds with bounded sectional curvature, upper diameter bound, converge to a metric space X with $\dim(X) = n - 1$, then X has the structure of a Riemannian orbifold. With these results in mind it seems reasonable that one should try to understand clearly the geometry of orbifolds. Orbifolds are a nice class of singu-