

Spectral Properties of Smooth Perturbations of Normal Operators with Planar Lebesgue Spectrum

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ABSTRACT. Smooth perturbations of normal operators whose spectral measure is absolutely continuous with respect to the planar Lebesgue measure are studied. We introduce a functional model of such operators which involves special Sobolev spaces defined by means of $\bar{\partial}$ -operator and the perturbation matrix (operator) function. Spectral division problems and questions of similarity and quasisimilarity are considered. It is shown that the zero set of the perturbation determinant coincides with the spectral singularity set defined in an invariant way. The perturbed operator is similar to the unperturbed one if and only if the perturbation determinant vanishes nowhere.

0. Introduction. Let N be a normal operator on a separable Hilbert space whose scalar spectral measure is absolutely continuous with respect to the planar Lebesgue measure. The paper is devoted to investigation of the spectral nature of the operator

$$(1) \quad T = N + K,$$

where K is a sufficiently “smooth” compact operator. We introduce the perturbation operator-function $\Psi(\lambda)$ of T defined everywhere in \mathbb{C} and establish the following *quotient model for T* : T is similar to the multiplication operator M_λ by the independent variable λ on the quotient space of a kind of vector anisotropic Sobolev space by the subspace of vector-valued functions of the form $\Psi(\lambda)a(\lambda)$, where a is analytic. Various consequences of this representation for the spectral theory of the operators under consideration are discussed.

The operators of the form (1) are obvious analogs of smooth perturbations of self-adjoint operators that have been studied intensively. General self-adjoint perturbations of self-adjoint operators are, in fact, the subject of the scattering theory. The specifics of smooth self-adjoint perturbations (the behavior of the