

# *Holomorphic Currents*

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**ABSTRACT.** In this paper we introduce a new method for constructing of plurisubharmonic functions. This method is applied to the construction of polynomial hulls, boundary problems of plurisubharmonic functions, the theory of capacities of boundary sets and to solving the degenerate Monge–Ampère equation.

**1. Introduction.** In this paper we study the space of plurisubharmonic (psh) functions, which is the closure in  $L^1_{\text{loc}}$  of the convex cone generated by functions  $\log|f(z)|$ , where  $f(z)$  is a holomorphic function. This definition explains the importance of psh functions for complex analysis as was demonstrated by the theory of holomorphic functions in one variable (see [24]). In this case one has the advantage of potential theory which allows us to split any subharmonic function  $u$  into the sum of a harmonic function (which is completely determined by the boundary values of  $u$ ) and a negative subharmonic function (which depends only on the Laplacian  $\Delta u$ ). Each of these functions has an integral representation which can be used for applications. Moreover, this splitting allows us to study separately harmonic functions and potentials of  $\Delta u$ .

The main goal of this paper is to extend methods of potential theory to the multi-dimensional case. A function  $u$  is plurisubharmonic if for every holomorphic mapping  $f$  of the unit disk  $U$  into  $\mathbb{C}^n$  the function  $u_f = u \circ f$  is subharmonic. Thus, every  $f$  defines the measure  $\mu_f = \Delta u_f$  on  $U$  and the measure  $\nu_f = u_f|_S$  on  $S$ . We may consider them as one measure  $\mu_f$  on the closure  $\bar{U}$  of  $U$ . Using the integral representation for  $u_f$ , we can recover  $u_f$  if  $\mu_f$  is known. Consequently, we can find  $u$  because  $u(z) = u_f(0)$  if  $f(0) = z$ . This simple idea was frequently used to get some information about  $u$ , but it was never used to construct psh functions. An arbitrary measure on  $\bar{U}$  determines uniquely, due to the integral representation, a subharmonic function, and this gives a lot of flexibility to the theory. If we choose measures  $\mu_f$  arbitrarily in the multi-dimensional case, then it could happen that  $f(0) = g(0)$  for two holomorphic mappings  $f$  and  $g$  of  $U$  but  $u_f(0) \neq u_g(0)$ . Nevertheless, we can use this idea to construct new functions, using the following method. Given a measure  $\mu_f$  on  $\bar{U}$ , we recover a subharmonic function  $u_f$  and then take the infimum of  $u_f(0)$  over all holomorphic mappings