

# *A Relation Between the Size of a Domain and the Solvability of the Dirichlet Problems for Semilinear Elliptic Equations on the Domain*

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**ABSTRACT.** We establish a relationship between the solvability of Dirichlet problems for semilinear elliptic equations on a domain and the measure of the set where the nonlinear term is positive. Given a nonlinear term, an upper bound for volumes of domains on which we can solve Dirichlet problems for the semilinear equation is obtained. The upper bound can be estimated explicitly by data depending only on the nonlinear term. Consequently, various kinds of existence results for Dirichlet problems for semilinear elliptic equations are proved. In our results, the corresponding functionals to Dirichlet problems may not satisfy the Palais-Smale condition.

**1. Introduction and Results.** Let  $\Omega$  be a bounded domain in  $R^n$ ,  $n \geq 2$ . We are concerned with the solvability of Dirichlet problems for semilinear elliptic equations

$$(P_0) \quad \begin{cases} -\Delta u = f(x, u) & \text{in } \Omega; \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

where  $f(x, u)$  is a local Hölder continuous function defined on  $\bar{\Omega} \times R$ .

Consider the functional corresponding to  $(P_0)$ ,

$$J_0(u) = \frac{1}{2} \int_{\Omega} |\nabla u|^2 dx - \int_{\Omega} F(x, u) dx, \quad u \in H_0^1(\Omega),$$

where  $F(x, t) = \int_0^t f(x, s) ds$ .

We shall establish new existence results for  $(P_0)$  for two types of nonlinear terms  $f(x, t)$ . We first consider those  $f(x, t)$  for which the size of set of  $x$ ,