

# *Estimates on the $(n - 1)$ -dimensional Hausdorff Measure of the Blow-up Set for a Semilinear Heat Equation*

J. J. L. VELÁZQUEZ

ABSTRACT. We consider the following problem

$$\begin{cases} u_t = \Delta u + u^p, & x \in \mathbb{R}^N, t > 0 \\ u(x, 0) = u_0(x), & x \in \mathbb{R}^N \end{cases}$$

where  $1 < p < (N + 2)/(N - 2)$ , and  $u_0(x)$  is a continuous nonnegative and bounded function. It is shown here that for any blowing-up solution which is different from the uniform one  $u_T(x, t) = ((p - 1)(T - t))^{-1/(p-1)}$  the  $(N - 1)$ -dimensional Hausdorff measure of the blow-up set is bounded in compact sets of  $\mathbb{R}^N$ .

**1. Introduction.** This paper deals with the following problem :

$$(1.1) \quad u_t = \Delta u + u^p \quad \text{when } x \in \mathbb{R}^N (N \geq 1), t > 0, 1 < p < \frac{N + 2}{N - 2},$$

$$(1.2) \quad u(x, 0) = u_0(x) \quad \text{when } x \in \mathbb{R}^N,$$

where  $u_0(x)$  is continuous, nonnegative and bounded. It is well known that solutions of (1.1), (1.2) may develop singularities in a finite time. In particular, a solution  $u(x, t)$  is said to blow up in a finite time  $T$  if  $u(x, t)$  satisfies (1.1), (1.2) in  $S_T = \mathbb{R}^N \times (0, T)$ , and

$$\limsup_{t \uparrow T} \left( \sup_{x \in \mathbb{R}^N} u(x, t) \right) = +\infty.$$

A point  $x_0$  is called a blow-up point if there exists sequences  $\{x_n\}, \{t_n\}$  such that  $\lim_{n \rightarrow \infty} x_n = x_0$ ,  $\lim_{n \rightarrow \infty} t_n = T$  and  $\lim_{n \rightarrow \infty} u(x_n, t_n) = +\infty$ . The set  $S$  consisting of all the blow-up points is termed the blow-up set. We refer to [BBE], [B], [FM], [L], [W] for earlier results in blow-up problems. Throughout this paper, we shall assume that:

$$(1.3a) \quad u(x, t) \leq M(T - t)^{-1/(p-1)}$$

for any  $x \in \mathbb{R}^N$ ,  $t < T$  and some constant  $M > 0$ .