

Atomic Correspondences

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ABSTRACT. If H is a correspondence between two von Neumann algebras M and N , and if \bar{H} denotes its conjugate, we study the property for the composed M - M correspondence $H \otimes_N \bar{H}$ to contain the identity M - M correspondence, in which case we say that H is left atomic. We are, in particular, interested in the situation where $N \subset M$ and H is the standard Hilbert space $L^2(M)$ viewed as an M - N bimodule. We get a notion, called atomic inclusion, usually weaker than that of finite index inclusion. When α is a continuous action of a locally compact group G on M , we study conditions under which the inclusions $M^G \subset M$ and $M \subset M \rtimes_\alpha G$ are atomic. As an application, we get comparison results on the types of M , M^G and $M \rtimes_\alpha G$.

Introduction. In order to define property T for von Neumann algebras, A. Connes introduced, several years ago, the key concept of correspondence between two von Neumann algebras M and N ([C2], [CJ]), pointing out that it plays the same role as a representation for a group. Later on, this study was developed by S. Popa [Po], and the recent works of A. Ocneanu and S. Popa highlight all the richness of such a structure.

Recall that a correspondence from M to N is a Hilbert space which is an M - N bimodule, with commuting normal actions. There are notions of trivial and coarse correspondences, respectively similar to trivial and regular representations for a group. Composition, induction and adjointness may be defined in a natural way for correspondences. Finally, if G is a locally compact group, and if $\mathcal{L}(G)$ denotes the von Neumann algebra generated by the left regular representation of G , a correspondence from $\mathcal{L}(G)$ to $\mathcal{L}(G)$ may be canonically associated with each representation of G . Thus, it is clear that a systematic investigation of the analogy between both notions of correspondence and group representation will give new insights in the von Neumann algebra theory. This point of view was successfully developed in [CJ] and [Po] for the study of rigidity and amenability problems.