

Positive Solutions of the Prescribed Mean Curvature Equation in \mathbf{R}^N

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Introduction. Our principal objective is to establish the existence of bounded positive classical solutions of the quasilinear elliptic equations

$$(1.1) \quad Lu = \lambda p(x)u^\gamma, \quad x \in \mathbf{R}^N, \quad N \geq 3,$$

where γ, λ are positive constants and L is the prescribed mean curvature operator, defined by

$$Lu = -\operatorname{div} \left[\frac{\nabla u}{\sqrt{1 + |\nabla u|^2}} \right].$$

Standing hypotheses are $0 \neq p \in C^1(\mathbf{R}^N, \mathbf{R}_+)$, where $\mathbf{R}_+ = [0, \infty)$, $(\nabla p)(x)$ is bounded in \mathbf{R}^N and $p(x) = O(|x|^{-b})$ as $|x| \rightarrow \infty$ for some constant $b > 1$. It is *not* required that p be radially symmetric. Under additional conditions on γ, p , positive solutions u_λ of (1.1) will be obtained which are asymptotic to prescribed positive constants as $|x| \rightarrow \infty$ for all λ in some positive interval, $0 < \lambda \leq \lambda^*$.

As far as we are aware, no results have been obtained to date for nonradial equations (1.1) in \mathbf{R}^N . If p is radially symmetric and satisfies a suitable decay condition at ∞ , Kusano and Swanson [6] recently found radial solutions of several types in \mathbf{R}^N . Equations of type $Lu = f(u)$ were studied in depth by Atkinson et al [1], Franchi et al [2,3], Ni and Serrin [7–9], and Peletier and Serrin [10]; existence and nonexistence theorems for decaying radial solutions were proved under various sign conditions and/or growth conditions on $f(u)$. Our procedure consists of the development of a “barrier method” for (1.1). The problem then reduces to the construction of subsolutions and supersolutions of (1.1) with appropriate asymptotic behavior. The main difficulty in dealing with nonradial equations (1.1) is the lack of suitable *a priori* gradient bounds. To overcome this, we first establish a local gradient bound for a sequence of “approximate solutions” for local problems on balls of radii $n = 1, 2, \dots$. This bound yields local compactness properties from which a standard diagonalization argument can be applied to obtain a solution of (1.1) in \mathbf{R}^N . Our methods are applicable to more general quasilinear equations

$$-\operatorname{div}[g(|\nabla u|)\nabla u] = f(x, u), \quad x \in \mathbf{R}^N,$$