

The Spectrum of the Cahn-Hilliard Operator for Generic Interface in Higher Space Dimensions

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1. INTRODUCTION

The nonlinear Cahn-Hilliard equation:

$$\begin{cases} u_t = \Delta(-\varepsilon^2 \Delta u + F'(u)), & x \in \Omega, \\ \frac{\partial u}{\partial n} = \frac{\partial}{\partial n}(-\varepsilon^2 \Delta u + F'(u)) = 0, & x \in \partial\Omega, \end{cases}$$

has been extensively studied in the last few years. Most of the interest stems from the fact that (1.1), a single equation, appears to capture, at least from the qualitative point of view, the rich phenomenology that can be observed in the phase separation of a binary alloy. We refer the reader to [C], [C-H], [F], [G1]–[G5], [Ca1], [Ca2], [P-F], [Pe], [B-F2] and to the references therein for information on the physical background, derivation of the equation, and discussion of related equations, and to [E-Fr], [McK], [F-G], [Ey] for numerical studies of this or other related equations. In these works it is shown that remarkably different nonlinear phenomena like nucleation, spinodal decomposition, formation and dynamics of fronts can be described by appropriate solutions of the Cahn-Hilliard equation.

In (1.1), $\Omega \subset \mathbb{R}^n$ is taken to be a bounded smooth domain, the container of the alloy, $u(x, t)$ stands for the concentration at x at time t , F is the bulk free energy and is assumed to have two equal nondegenerate minima at $u = u_1$ and $u = u_2$ corresponding to two equally preferred phases. A typical choice is $F(u) = u^2(u^2 - 1)$. The term $-\varepsilon^2 \Delta u$ models the effect of surface tension. The parameter ε measures the importance of surface energy with respect to bulk free energy and it is taken $\ll 1$. The existence of two distinct stable phases with the same free energy together with the smallness of ε that allows for high gradients of u , suggest the existence of solutions of (1.1) with sharp interfaces (between u_1 and u_2) that may be stationary (layered equilibria) or slowly evolving (moving fronts). In fact, the existence of solutions of this kind has been shown by several authors [C-G-S], [Mod], [Lu-Mod], [Ster], [G-M], [A-B-F1], [Bro-H], [Gr], [K-X].