

Invariant Manifolds and Inertial Forms for Parabolic Partial Differential Equations

JEONGSEON BAEK, HYEONG-KWAN JU & MINKYU KWAK

ABSTRACT. In this paper we consider a semilinear parabolic partial differential equations of the form

$$(1) \quad \frac{\partial u}{\partial t} = \Delta u + F(x, u, \nabla u) \quad \text{in } \Omega \subset \mathbf{R}^N$$

with periodic boundary condition.

Under suitable assumptions on F and Ω , we show (by applying the inertial manifold theory) that there exists a finite dimensional smooth manifold \mathcal{M} which attracts exponentially all solutions of (1).

By considering the dynamics on \mathcal{M} , we will then find a finite-dimensional system of ordinary differential equations that describes completely the long-time dynamics of (1).

1. Introduction. Let $\Omega = (0, L_1) \times (0, L_2)$. We are concerned with the asymptotic dynamics of a certain type of semilinear parabolic differential equation, namely,

$$(1.1) \quad u_t = \Delta u + F(x, u, \nabla u) \quad \text{in } \Omega.$$

We assume u is scalar and, for boundary conditions, we take a space-periodic boundary condition even though most of all the results are applicable to the Dirichlet and Neumann boundary conditions. We can also extend our results to parabolic systems.

Let $F(x, u, p)$ be a sufficiently smooth function defined on $\Omega \times \mathbf{R} \times \mathbf{R}^2$. We assume that (1.1) admits a dissipative dynamical system (see Hale (1988), Temam (1988)). More precisely, throughout this paper we assume that the unique regular solution of (1.1) exists for all time $t \geq 0$ provided the initial data is in $H_{\text{per}}^2(\Omega)$ and that the solution becomes sufficiently regular for $t > 0$ —see Henry (1981). In particular, we assume that the global attractor \mathcal{A} and an absorbing ball \mathcal{B} of radius ρ_0 exist in $H_{\text{per}}^4(\Omega)$, where

$$H_{\text{per}}^m(\Omega) = \left\{ u : u = \sum_{k \in \mathbf{Z}^2} c_k e^{2\pi i(k_1 x_1 / L_1 + k_2 x_2 / L_2)}, \bar{c}_k = c_{-k}, \sum_{k \in \mathbf{Z}^2} |k|^{2m} |c_k|^2 < \infty \right\}.$$