Free Convolution of Measures with Unbounded Support

HARI BERCOVICI & DAN VOICULESCU

To Ciprian Foias on his sixtieth birthday
La Mulţi Ani!

1. Introduction. In the noncommutative probability theory of free products, addition and multiplication of free random variables corresponds with the additive, and respectively multiplicative, convolution of the corresponding probability distributions; cf. [12] and [13]. Carrying further the analogy with independence and convolution in usual probability theory, the infinitely divisible compactly supported probability measures where studied in [12] and [2]. To deal with the analogous problems for unbounded free random variables requires an extension of free convolutions to measures with unbounded support. The extension to unbounded supports is unavoidable in the study of stable laws for free convolution which we undertake here. Since free convolution (for measures with bounded support) was computed using a transformation which is the free analogue of the logarithm of the Fourier transform, we also have to deal with the analytical problems involved in using the same transform for measures with unbounded support. In a certain sense this note is a step in an effort to refine and extend one-dimensional free harmonic analysis.

We would like to mention that the extension of the additive free convolution to measures with unbounded support but with finite variance was done by Maassen [9]. His work relies on developing the analytic function machinery directly from the unbounded variables. Our approach is different; we first establish continuity and monotonicity properties for free convolution working in the context of unbounded operators affiliated with finite von Neumann algebras. The analytical problem becomes that of an extension by continuity. Some technical facts from [9] provided useful inspiration for some of the analytic questions arising in our approach, as will be seen below.

We are indebted to Beresford Parlett who provided us with a reference for Theorem 3.6 (in fact he first showed us a much shorter proof of that result than the one we had found ourselves).

Indiana University Mathematics Journal ©, Vol. 42, No. 3 (1993)