

Algorithms for Optimizing over Analytic Functions

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ABSTRACT. Let H_N^∞ be the space of C^N -valued measurable functions on the unit circle ∂D that extend to a uniformly bounded analytic function on the unit disk D . A fundamental optimization problem in frequency domain design where stability is the key constraint are:

(OPT $^\infty$) Given a map $\Gamma : \partial D \times C^N \rightarrow R^+$ and $E \subset H_N^\infty$, minimize $\sup_{e^{i\theta} \in \partial D} \Gamma(e^{i\theta}, f(e^{i\theta}))$ over $f \in E$ and find minimizers f^* in E if they exist.

The main contribution of the article is to present an algorithm for solving OPT $^\infty$ when Γ is smooth and E is the set of functions in H_N^∞ that are smooth on ∂D . The algorithm is quadratically convergent and is conceptually different from existing algorithms. In this article we prove that the algorithm is (locally) quadratically convergent to solutions to OPT $^\infty$ and illustrate practical performance of the algorithm with results of computer runs.

Another problem studied in this article is:

(OPT 1) Given a map $\Gamma : \partial D \times C^N \rightarrow R^+$ and $E \subset H_N^\infty$, minimize $\int_0^{2\pi} \Gamma(e^{i\theta}, f(e^{i\theta})) d\theta / 2\pi$ over $f \in E$ and find minimizers f^* in E if they exist.

We derive the optimality conditions that characterize local solutions to OPT 1 when $E = A_N$, the set of functions in H_N^∞ that are continuous on ∂D . An algorithm for solving OPT 1 when Γ is smooth and E is the set of smooth H_N^∞ functions is