

# *On Invariant Curve Evolution and Image Analysis*

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**ABSTRACT.** This paper deals with the mathematical theory of invariant curve evolution. We present a high-level procedure for the formulation of *geometric heat flows* which are invariant with respect to a given Lie group. This approach is based on the classical theory of differential invariants. The affine group is then analyzed in detail. Indeed, we give a rather complete description of the properties of the *affine geometric heat equation*. We moreover extend the results of [38] from the convex to the nonconvex case. The paper concludes with a summary of recent applications of curve evolution theory to image analysis.

*This paper is dedicated to a great mathematician and a great friend Ciprian Foias on the occasion of his 60th birthday. La multi ani, Ciprian.*

**1. Introduction.** The theme of this paper is two-fold: invariant curve evolution theory (and especially invariant nonlinear diffusion equations) on the one hand, and the employment of such a methodology for new algorithms in image processing on the other. Curve evolution theory has recently become a major topic of research, and indeed has appeared in such diverse fields as differential geometry [12, 17, 18, 38, 48], parabolic equations theory [4, 5], numerical analysis [29], computer vision [21, 22, 39], viscosity solutions of Hamilton-Jacobi equations [6, 10, 13, 16], and image processing [3]. In particular, evolution equations which are geometric non-linear versions of the classical heat equation have received much attention, since these equations have both theoretical and practical importance. In this paper, we would like to emphasize the importance of the theory to mathematical problems in computer vision and image processing.

In general, two steps are involved in the research of curve evolution. The first one consists of the search for the evolution equation which best models the given problem. Some of the physical phenomena that can be modeled in this