

The Bifurcation of Periodic Solutions of the Nonlinear Telegraph Equation and Infinite Dimensional Singularity Theory

MELVYN BERGER & GUOZHANG SUN

1. Introduction. In this article, we consider the bifurcation of T -periodic solutions of the boundary value problem of the nonlinear telegraph equation:

$$(1) \quad \begin{aligned} u_{tt} + \alpha u_t + Lu - f(u) &= g(x, t) && \text{in } \Omega \times R, \\ u &= 0 && \text{on } \partial\Omega \times R, \end{aligned}$$

obtained for a given smooth forcing term $g(x, t)$, which is T -periodic in t . Here $\alpha \neq 0$ is a constant and the domain $\Omega \subset R^n$ is an arbitrary bounded domain with smooth boundary $\partial\Omega$. The operator L is a uniformly elliptic, formally self-adjoint second order differential operator defined on Ω .

The function f generating the nonlinearity is specialized in two interesting cases:

- (i) f is a C^2 -convex function such that $f''(0) > 0$ and $f(0) = 0$, satisfying the asymptotic conditions:

$$0 < \lim_{s \rightarrow -\infty} \frac{f(s)}{s} < \lambda_1; \quad \lambda_1 < \lim_{s \rightarrow +\infty} \frac{f(s)}{s} < \lambda_2$$

where λ_1, λ_2 are the lowest two eigenvalues of L .

- (ii) $f(s) = \lambda s - b(s)$ where λ is a parameter and the function $b : R \rightarrow R$ is a C^3 function such that the derivatives of b satisfy:

$$\begin{aligned} b'(s) &> 0 && \text{for every } s \neq 0, \\ b^{(j)}(0) &= 0 && \text{for } j = 0, 1, 2; \\ b^{(3)}(s) &\geq b^{(3)}(0) > 0 && \text{for all } |s| < 1. \end{aligned}$$

Here the function b is given as $b(s) \sim s^3$ for $|s| < 1$ and $b(s) \sim s$ for $|s| \geq 1$.