

Manifolds of Slow Solutions for Highly Oscillatory Problems

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ABSTRACT. We consider stiff oscillatory systems of ordinary differential equations and ask for slowly varying solutions. As is well-known, if the initial data are chosen by the bounded derivative principle, then the solution varies slowly up to some order. In this paper we clarify the relation between the bounded derivative principle and the existence of a local or global slow manifold. The manifolds are obtained by solving a first-order system of partial differential equations. Also, we estimate the interaction between the fast and the slow scale for solutions which start off the slow manifold.

1. Introduction. In this paper we consider real dynamical systems of the form

$$(1.1) \quad \begin{aligned} \varepsilon y' &= (A(t) + \varepsilon C(v, y, t))y + f(v, t), \quad 0 < \varepsilon \leq \varepsilon_0, \\ v' &= g(v, y, t) \end{aligned}$$

where

$$y(t), f(v, t) \in \mathbb{R}^m, \quad v(t), g(v, y, t) \in \mathbb{R}^n, \quad A(t), C(v, y, t) \in \mathbb{R}^{m \times m}.$$

Our main assumption is as follows:¹

Assumption 1.1. For all $t \geq 0$,

$$(1.2) \quad A(t) + A^*(t) \leq 0, \quad \det A(t) \neq 0.$$

In typical applications, the matrix $A(t) = -A^*(t)$ will be skew-Hermitian; then for most initial data

$$v(0) = v_0, \quad y(0) = y_0$$

¹ With $\langle w, z \rangle = \sum_{\mu=1}^k w_\mu z_\mu$ and $|w| = \sqrt{\langle w, w \rangle}$ we denote the Euclidean innerproduct and norm of vectors, respectively. For Hermitian matrices B, C we write $B \leq C$ if and only if $\langle w, Bw \rangle \leq \langle w, Cw \rangle$ for all w .