

Meromorphic Functions with Small Ramification

A. EREMENKO

Abstract. Let f be a meromorphic function in the plane and $N_1(r)$ stands for the Nevanlinna counting function of critical points of f . We obtain a complete description of functions f of finite lower order with the property

$$N_1(r) = o(T(r)), \quad r \rightarrow \infty.$$

In particular, we show that this property is equivalent to

$$\sum_{a \in \bar{\mathbf{C}}} \delta(a, f) = 2.$$

1. Introduction. For a function f meromorphic in the complex plane \mathbf{C} we use the standard notations of Nevanlinna theory: $T(r, f)$, $N(r, f)$, $m(r, f)$, $\delta(a)$, etc. (See [20], [14].) Recall that the “ramification term”

$$N_1(r) = N(r, 1/f') + 2N(r, f) - N(r, f')$$

counts the multiple points of f . The second main theorem of Nevanlinna states that for arbitrary collection of distinct $a_1, \dots, a_q \in \bar{\mathbf{C}}$ we have

$$(1) \quad \sum_{k=1}^q m(r, a_k) + N_1(r) \leq 2T(r) + S(r),$$

where S is a small error term. Particularly for functions f of finite order $S(r) = O(\log r)$, $r \rightarrow \infty$, and from (1) it follows that

$$(2) \quad \sum_{k=1}^q \delta(a_k) + \vartheta \leq 2,$$