

Existence and Classification of Constant Mean Curvature Multibubbletons of Finite and Infinite Type

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0. Introduction. Complete surfaces of constant Gauss curvature (CGC) and also surfaces of constant mean curvature (CMC) were studied extensively in the 19th century. Much of this material may be found in the treatise of G. Darboux [D] and also the book of L. P. Eisenhart [Ei]. For an historical survey of the subject one may read the recent work of Melko and Sterling [MS1] and also the paper of Wente [W]. One method for constructing examples was to assume some extra geometric condition. Surfaces of revolution, surfaces with one family of planar curvature lines and surfaces with one family of spherical curvature lines were studied by Delaunay, Enneper and Dobriner respectively. Another method, due to Bäcklund, was recursively to construct new examples from old ones. These two methods complement one another and in fact naturally intersect, producing some especially beautiful examples.

Originally Bäcklund's transformation showed how to construct new CGC surfaces with Gauss curvature $K = -1$ from a given one (see Eisenhart [Ei] for the discussion). Bianchi [Bi] extended Bäcklund's method to the more difficult case of CGC surfaces with $K = +1$. In Sections 1 and 2 we review and carry out Bianchi's construction.

The deformations arising from these multisoliton solutions are geometrically simple and appear as bubbles spinning around, moving along and colliding on a flat cylinder. (Compare with the more subtle case of deforming tori [PS2].) For this reason we shall refer to our examples as multibubbletons (Figure 2). In Section 3 we give an informal computational discussion of the moduli space of doublebubbletons. In Section 4 we prove the existence of CMC surfaces which are constructed by adding infinitely many bubbletons. These examples produce new solutions to the sinh-Gordon equation which are not of finite (gap) type. Recently, similar results have been announced for the KdV equation [GKZ]. In Section 5 we prove that the total absolute curvature of Sievert's singlebubbleton (Figure 1) is always an integer multiple of 8π .