

# *Linearly Constrained Interpolation in Nest Algebras*

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ABSTRACT. A linearly constrained interpolation problem in nest algebras is solved. The result generalizes the well-known Arveson distance formula. Applications concern quasitriangular operators and a variant of the operator theoretic Corona theorem.

**1. The Main Result.** Let  $\mathbf{H}$  be a Hilbert space and  $\mathbf{N}$  a nest of orthogonal projections in  $\mathbf{B}(\mathbf{H})$  (the Banach algebra of bounded linear operators on  $\mathbf{H}$ ), i.e.,  $\mathbf{N}$  is a strongly closed, linearly ordered collection of projections on  $\mathbf{H}$ , containing 0 and the identity. Recall that the nest algebra corresponding to  $\mathbf{N}$  is defined as

$$\text{Alg } \mathbf{N} := \{A \in \mathbf{B}(\mathbf{H}) \mid P^\perp AP = 0 \text{ for all } P \in \mathbf{N}\},$$

where  $P^\perp = I - P$ . We refer to [D] as a reference on nest algebras.

Our main result is the following. The Banach space of bounded linear operators acting from the Hilbert space  $\mathbf{K}$  to  $\mathbf{H}$  is denoted by  $\mathbf{B}(\mathbf{K}, \mathbf{H})$ .

**Theorem 1.1.** *Let  $\mathbf{N}$  be a nest on a Hilbert space  $\mathbf{H}$  and  $B \in \mathbf{B}(\mathbf{H})$ ,  $S, T \in \mathbf{B}(\mathbf{K}, \mathbf{H})$  given operators. Then*

$$(1.1) \quad \inf\{\|A\| \mid A \in B + \text{Alg } \mathbf{N}, AS = T\} \\ = \inf\left\{k \mid \begin{bmatrix} S^*P^\perp S & (T^* - S^*PB^*)P^\perp \\ P^\perp(T - BPS) & k^2I - P^\perp B P B^* P^\perp \end{bmatrix} \geq 0 \text{ for all } P \in \mathbf{N}\right\}.$$

*Proof.* In order to obtain the inequality  $\geq$ , let  $A \in B + \text{Alg } \mathbf{N}$  satisfy  $AS = T$  and denote  $k = \|A\|$ . By Lemma 4.1 in [BW],

$$\begin{bmatrix} I & S & A^*/k \\ S^* & S^*S & T^*/k \\ A/k & T/k & I \end{bmatrix} \geq 0.$$