

# *Existence of a Class of Symmetric Rotating Spiral Waves on Finite Disc Domains in Excitable Media*

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**ABSTRACT.** We prove the existence of symmetric spiral wave solutions to Fife's free boundary problem on finite disc domains in  $\mathbb{R}^2$ , with the Dirichlet boundary condition for the controller variable and a suitable boundary condition for the spiral shape function, under the condition that the domain radius is relatively small with respect to the excitability of the media. We also consider a slightly extended version of this free boundary problem with the Neumann boundary condition. We use the elliptic  $W^{2,p}$  theory, maximum principle, and a-priori estimates. Without the condition for the domain radius, it is possible that the spiral curve would develop singularities. We show that they are cusp-like singular points whenever isolated, and the spiral curve is absolutely continuous around them. We also point out the obstructions to constructing spiral waves on  $\mathbb{R}^2$ .

**1. Introduction.** Rotating spiral waves in excitable media, as one of the most interesting examples of spatial pattern formation in nonequilibrium systems, have been the focus of much research in various areas including biology, chemistry, cardiology, and applied mathematics. For a review of recent developments, see [21] and the references therein. The most studied problem may well be the formation of spirals in the Belousov-Zhabotinsky reaction, among other excitable media such as the heart muscle or social amoeba colonies. A commonly used phenomenological model for qualitatively describing such spirals is the following system of reaction-diffusion equations with two-diffusing species:

$$(1.1) \quad \begin{cases} u_t = \varepsilon \Delta u + \frac{1}{\varepsilon} f(u, v) \\ v_t = \varepsilon D \Delta v + g(u, v), \end{cases}$$

where  $D > 0$ ,  $0 < \varepsilon \ll 1$ ,  $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ ,  $(x, y) \in \mathbb{R}^2$ , and  $f, g$  are