

The Bergman Kernel Function and the Ahlfors Mapping in the Plane

YOUNG-BOK CHUNG

ABSTRACT. An interesting formula relating the exact Bergman kernel and the Ahlfors mapping function on a smooth bounded domain in the plane is found.

1. Introduction. One main theme of the work of Bergman and Schiffer has been that many of the classical objects of potential theory and conformal mapping are expressible in terms of kernel functions. A prime example of this phenomenon is given by Garabedian's theorem which states that the Ahlfors map associated to a finitely connected domain in the plane is a quotient of kernel functions associated to the Szegő projection. Suppose that Ω is a bounded domain with C^∞ smooth boundary in the plane. It is well known (see [2, 5, 6]) that if Ω is simply connected, then the Bergman kernel function $B(z, \zeta)$ and the Riemann mapping function f_ζ associated to Ω with the usual normalization $f_\zeta(\zeta) = 0$, $f'_\zeta(\zeta) > 0$, are related to the Szegő kernel function $S(z, \zeta)$ via the identities $B(z, \zeta) = 4\pi S(z, \zeta)^2$ and $f'_\zeta(z) = 2\pi S(z, \zeta)^2 / S(\zeta, \zeta)$. Hence in the simply connected case, we have the following relationship between the Bergman kernel function and the Riemann mapping function:

$$B(z, \zeta) = 2S(\zeta, \zeta)f'_\zeta(z).$$

It is natural to ask the same sort of question on the multiply connected domains. Observe that if Ω is simply connected, the exact Bergman kernel and the Ahlfors mapping are equal to the Bergman kernel and the Riemann mapping, respectively.

The main result of this paper improves upon a result of Schiffer relating the Bergman kernel function to the Neumann function associated to a finitely connected domain in the plane. We shall prove a relationship between the Bergman kernel function and the Ahlfors mapping function in the multiply connected case. And then we remark about the behavior of the derivative of the Ahlfors mapping. In particular, we shall show that the derivative of the Ahlfors map has