

# *Complex Stable Manifolds of Holomorphic Diffeomorphisms*

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**Abstract.** For any holomorphic diffeomorphism  $f$  on  $\mathbb{C}^n$  with an  $f$ -invariant compact subset  $K$  and any  $f$ -invariant probability Borel measure  $\mu$  on  $K$ , we give the complex versions of the Oseledec multiplicative ergodic theorem and the Pesin stable manifold theorem. If  $f$  is a finite composition of complex Hénon mappings of  $\mathbb{C}^2$ ,  $K$ ,  $J^+$  and  $J^-$  are  $f$ -invariant sets defined in the introduction, and  $\mu$  is the equilibrium measure introduced by Bedford, Smillie and Sibony on  $K$ , it is proved that for  $\mu$  a.e.  $p \in K$ , the stable/unstable manifolds are immersed holomorphic copies of  $\mathbb{C}$ , and they are contained in  $J^+/J^-$ .

**0. Introduction.** Pesin [P1, P2] established a remarkable stable manifold theorem for  $C^{1+\varepsilon}$  ( $\varepsilon > 0$ ) diffeomorphisms  $f$  of compact Riemannian manifolds  $M$  as following: Given an  $f$ -invariant probability Borel measure  $\mu$ , then for  $\mu$ -a.e.  $p$ , the stable set at  $p$

$$W^s(p, f) = \left\{ q \in M : \limsup_{n \rightarrow \infty} \frac{1}{n} \log d(f^n(q), f^n(p)) < 0 \right\}$$

is an immersed Euclidean space.

The unstable set at  $p$ , denoted by  $W^u(p, f)$ , is defined analogously by using  $f^{-1}$  instead of  $f$ . For simplicity, we only consider the stable sets.

Let  $f$  be a holomorphic diffeomorphism of  $\mathbb{C}^n$  and  $K$  an  $f$ -invariant compact subset. In this paper we will study the properties of stable manifolds of  $f$ . In Section 1 we give the statements of the complex version of the Oseledec Multiplicative Ergodic Theorem (Theorem 1.1) and the complex version of the Pesin Stable Manifold Theorem (Theorems 1.2 and 1.3). In Section 2 we apply the