

On the Gauss Map of Complete Minimal Surfaces with Finite Total Curvature

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ABSTRACT. We prove that if M is an immersed complete minimal surface in \mathbb{R}^3 , and the total curvature of M is larger or equal to -20π , then the Gauss map of M can only miss at most two values in the Riemann sphere $\Sigma = \mathbb{C} \cup \{\infty\}$. In general, we prove that if all the ends of M are embedded, and the Gauss map of M misses three values, then the number of flat ends of M is larger than 2. Finally, we give an example of a properly immersed singly periodic minimal surface whose Gauss map misses three values.

1. Introduction. The study of the global properties of complete, immersed minimal surfaces in \mathbb{R}^3 , especially the properties of their Gauss map, has achieved many important results and also given rise to many problems in recent decades. Among them a very interesting problem is the following: if $X : M \rightarrow \mathbb{R}^3$ is a complete non-planar minimal immersion, and $N : M \rightarrow S^2 \cong \mathbb{C} \cup \{\infty\} = \Sigma$ its Gauss map, how many values can N miss?

In 1961 Osserman [13] proved that N can miss at most a set of points of logarithmic capacity zero. In 1981 Xavier [19] proved that N can miss at most 6 values. In 1988 Fujimoto [6] proved that N can miss at most 4 values. This is the best possible number, since there are a lot of examples of complete minimal immersions whose Gauss map misses 4 values. An example is Scherk's surface, which is even embedded. See [15] (page 72, Theorem 8.3) for other examples. However, all of these examples have infinite total Gauss curvature. This is not a surprise, since in 1964 Osserman [14] proved that if M has finite total curvature, then N can miss at most 3 values.

The catenoid has finite total curvature of -4π and its Gauss map misses 2 values. There is no known example of a complete immersed minimal surface of finite total curvature whose Gauss map misses 3 values. Hence either 2 or 3 is the maximal number of values that may be omitted. Since Osserman posed this problem in 1967, many years have passed without any progress. To look