

A Diffusion-convection Equation in Several Space Dimensions

MIGUEL ESCOBEDO, JUAN LUIS VÁZQUEZ & ENRIKE ZUAZUA

ABSTRACT. We study the large-time behaviour of nonnegative solutions $u(x, t)$ of the diffusion-convection equation $u_t = \Delta u - \mathbf{a} \cdot \nabla(u^q)$, defined in the whole space \mathbf{R}^N for $t > 0$, with initial data $u_0 \in L^1(\mathbf{R}^N)$. The direction \mathbf{a} is assumed to be constant.

We concentrate in the exponent range $1 < q < (N + 1)/N$ and show that for very large times the effect of diffusion is negligible as compared to convection in the direction \mathbf{a} , while in the directions orthogonal to \mathbf{a} the motion is explained by diffusion. More precisely, the asymptotic behaviour of the solutions to (1) is given by the fundamental entropy solutions of the reduced equation (2) $u_t = \Delta' u + \mathbf{a} \cdot \nabla(u^q)$, where Δ' denotes the $(N - 1)$ -dimensional Laplacian in the hyperplane orthogonal to \mathbf{a} . Existence and uniqueness of such special solutions, which have a selfsimilar form, is proved here, previous to establishing the asymptotic convergence.

Such a phenomenon does not occur for $q = 1$ or for $q \geq (N + 1)/N$.

Introduction

I. The aim of this paper is to study the large-time behaviour of the nonnegative solutions of the Cauchy problem for the nonlinear diffusion-convection equation

$$(0.1) \quad u_t = \Delta u - \mathbf{a} \cdot \nabla(u^q),$$

posed in $Q = \mathbf{R}^N \times (0, \infty)$ with initial data

$$(0.2) \quad u(\cdot, 0) = u_0(\cdot) \quad \text{in } \mathbf{R}^N.$$

We assume for convenience in the study that the vector giving the convection direction is constant, $\mathbf{a} \in \mathbf{R}^N$. We also assume that u_0 is nonnegative and integrable, $u_0 \in L^1(\mathbf{R}^N)$. The exponent q is assumed to be larger than 1. In