

Compactness Results for Complete Metrics of Constant Positive Scalar Curvature on Subdomains of S^n

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ABSTRACT. In this paper we prove that the set of metrics conformal to the standard metric on $S^n \setminus \{p_1, \dots, p_k\}$, where $\{p_1, \dots, p_k\}$ is an arbitrary set of $k > 1$ points, is compact in the $C^{m,\alpha}$ topology, provided the Dilational Pohožaev invariants are bounded away from zero. Such metrics are necessarily complete on $S^n \setminus \{p_1, \dots, p_k\}$ and asymptotic, at each singular point, to one of the periodic, spherically symmetric solutions on the positive n -dimensional half cylinder, which are known as Delaunay metrics. The Pohožaev invariants are defined at each singular point, and their dilational component is shown to be equal to a dimensional constant times the Hamiltonian energy of the asymptotic Delaunay metric. We also address the issue of convergence when the invariants tend to zero for a proper subset of the singular points.

1. Introduction. In this paper we prove a compactness theorem for certain complete metrics of constant positive scalar curvature on subdomains of S^n . The metrics we consider are conformal to the round metric, on a domain $\Omega \subset S^n$ that is the complement of a finite set of points $\Lambda = \{p_1, \dots, p_k\}$, for $k \geq 2$. These metrics are solutions to the singular Yamabe problem. In its most general form, this problem seeks metrics of constant scalar curvature that are conformal to a background metric g_0 on a compact Riemannian manifold M and which have a specified closed set $\Lambda \subset M$ as their singular set.

If we write $g = u^{4/(n-2)}g_0$ for a smooth positive function u on $\Omega = M \setminus \Lambda$, then the condition that the scalar curvature $R(g)$ of g be equal to a constant K is equivalent to u satisfying

$$L_{g_0}u + c(n)Ku^{(n+2)/(n-2)} = 0,$$

on Ω , where $c(n) = (n-2)/(4(n-1))$ and $L_{g_0} = \Delta_{g_0} - c(n)R(g_0)$ is the conformal Laplacian with respect to the metric g_0 .