

The Helmholtz Equation and Transmission Problems with Lipschitz Interfaces

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ABSTRACT. We use the method of layer potentials to study a transmission problem for the Helmholtz equation in Lipschitz domains. Following the approach for the case of smooth interface, we propose as solution an ansatz of combinations of single and double layer potentials. In the case of Lipschitz interfaces, however, the lack of smoothness introduced some additional difficulties. These are overcome through the use of harmonic analysis techniques together with a careful study of the properties of the normal derivative of the double layer potential and the spectrum of the trace of the double layer potential. We also extend some known results for the Dirichlet and Neumann problems for the Helmholtz equation to a larger set of wave numbers.

1. Introduction. The study of transmission problems and other boundary value problems for the Helmholtz equation, $\Delta u + k^2 u = 0$, has received a lot of attention. These problems arise in connection with the analysis of the diffraction of a time harmonic acoustic wave by an obstacle. They also appear, in the vector valued case, in the study of Maxwell equations and the scattering of electromagnetic waves. In the case of smooth domains, these types of problems have been successfully studied using the classical method of layer potentials. In particular, we mention the work by Kress and Roach [21], whose approach to transmission problems we shall follow. There are a number of references given there which detail some of the history of transmission problems up to the publication of [21]. Other references can be found in the book [8] which gives a very complete description of the use of layer potentials for boundary value problems for the Helmholtz equation in smooth domains. In this paper we treat domains which are separated by (a considerably rougher interface) a Lipschitz surface, and we want to consider boundary data in L^p or appropriate Sobolev spaces.