

# A Class of Fully Nonlinear Elliptic Equations and Related Functionals

XU-JIA WANG

ABSTRACT. In this paper we study the existence of classical solutions to a class of fully nonlinear elliptic equations determined by functions of the eigenvalues of the Hessian. We also deal with the corresponding functionals and obtain, among others, an inequality analogous to the Sobolev inequality  $\|u\|_{L^{2n/(n-2)}(\Omega)} \leq C\|Du\|_{L^2(\Omega)}$ .

**1. Introduction .** Let  $\Omega$  be a bounded smooth domain in  $\mathbb{R}^n$ . For a function  $u \in C^2(\Omega)$  we denote by  $\lambda = (\lambda_1, \dots, \lambda_n)$  the eigenvalues of the Hessian matrix  $(D^2u)$ . In this paper we deal with the existence of solutions to the problem

$$(1.1) \quad \begin{cases} S_k(D^2u) = f(x, u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

and the corresponding functional

$$(1.2) \quad H_k(u) = - \int_{\Omega} u S_k(D^2u) \, dx,$$

where

$$(1.3) \quad S_k(D^2u) = \sigma_k(\lambda) = \sum_{i_1 < \dots < i_k} \lambda_{i_1} \dots \lambda_{i_k}$$

is the  $k$ -th elementary symmetric function of the eigenvalues  $\lambda = (\lambda_1, \dots, \lambda_n)$ ,  $f(x, u)$  is a nonnegative smooth function.

Equations of this type, and some more general equations of the form

$$F(\lambda_1, \dots, \lambda_n) = f \text{ in } \Omega,$$