

# Hausdorff Dimension of Multiple Points of the $(N, d)$ Wiener Process

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**1. Introduction.** Let  $W^{N,d}$  denote the  $(N, d)$  Wiener process with values in  $d$ -dimensional Euclidean space, i.e.,

$$W^{N,d}(t, \omega) = (W_1(t, \omega), \dots, W_d(t, \omega)) \in R^d,$$

where  $t \in R_+^N$  and the component  $W_i$ 's are independent, separable Gaussian processes with zero means and covariance

$$EW_i(s)W_i(t) = \prod_{j=1}^N (s^j \wedge t^j), \quad 1 \leq i \leq d,$$

where  $s = (s^1, \dots, s^N)$ ,  $t = (t^1, \dots, t^N)$ . In this paper, for  $W^{N,d}$ , we get the following result:

**Theorem.** Let  $P_1, \dots, P_k \subset R^N$  be compact sets without common parts and  $\text{Int } P_i \neq \emptyset$  ( $1 \leq i \leq k$ ). Denote

$$M_k(P) = \{x \in R^d \mid \exists \underline{t} = (t_1, \dots, t_k) \in P = P_1 \times \dots \times P_k,$$

$$\text{such that } W^{N,d}(t_1) = \dots = W^{N,d}(t_k) = x\},$$

$$M_k = \{x \in R^d \mid \exists \underline{t} = (t_1, \dots, t_k) \in (R^N)^k$$

$$\text{such that } W^{N,d}(t_1) = \dots = W^{N,d}(t_k) = x\}.$$

If  $d \geq 2N$ ,  $2Nk \geq (k-1)d$ , then

$$\dim M_k(P) = 2Nk - (k-1)d \quad \text{almost surely,}$$

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where  $\dim A$  denotes the Hausdorff dimension of the set  $A$ .