

Universal Factorization of Positive Operators

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ABSTRACT. If a positive invertible operator A factors as $A = T^*T$ for invertible operators T in every nest algebra, then A must be scalar plus compact. If in addition, the operator T can always be chosen to be scalar plus compact, then A is scalar plus an element of the Mac'aeu ideal. This is a sharp converse to a result of Gohberg and Krein.

Introduction. The problem of factoring a positive invertible operator A as T^*T for some invertible element of a nest algebra goes back to Gohberg and Krein [6, 7]. They showed that every operator of the form $\lambda I + K$, where K belongs to the Mac'aeu ideal, has such a factorization with respect to every nest algebra. Moreover, the operator T is scalar plus compact. In this paper, we answer a question of Gohberg and Krein's by establishing the converse of this result: every operator which factors as T^*T for some $T = \lambda I + K$, K compact, in every nest algebra is necessarily of the form scalar plus Mac'aeu. In addition, we show that if the form of T is not restricted, it is still necessary that A be scalar plus compact if such a factorization exists for every nest.

Such factorizations have been pursued by many authors, Arveson [2], Larson [10], Power [12, 13], Pitts [11], etc. (see [4] for an overview). Larson showed that there is a strong connection between these factorization questions and the similarity theory of nests. In particular, he proved that a nest has the property that *every* positive invertible operator factors as T^*T for some invertible T in the nest algebra if and only if the nest is countable. He also showed that if the condition that T have an inverse in the nest algebra is dropped, then such a factorization always exists. This generalized an argument of Choi and Feintuch [3].

In this paper, \mathcal{H} will always denote a separable Hilbert space. We denote the bounded linear operators by $\mathcal{B}(\mathcal{H})$, and \mathcal{K} or $\mathcal{K}(\mathcal{H})$ will denote the ideal of compact operators. A nest \mathcal{N} is a family of closed subspaces of \mathcal{H} which forms