

On The Neumann Problem for Schrödinger Operators in Lipschitz Domains

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ABSTRACT. We consider the L^p -Neumann problem for the Schrödinger operator $-\Delta + V$ in the region above a Lipschitz graph. For $1 < p \leq 2$, it is shown that, under suitable conditions on the potential V , there exists a unique solution u such that the nontangential maximal function of ∇u is in L^p .

Introduction. In recent years, there has been considerable interest in boundary value problems in nonsmooth domains. For the Laplace's equation in Lipschitz domains, it was shown that the Neumann problem with data in L^p is uniquely solvable for $1 < p \leq 2$ (see [JK], [V], [DK]). In this paper, we consider the L^p -Neumann problem for the Schrödinger equation:

$$(0.1) \quad -\Delta u + Vu = 0 \quad \text{in } \Omega$$

where $V = V(X) \geq 0$ and $\Omega \subset \mathbb{R}^n$, $n \geq 3$, is the region above a Lipschitz graph. Under suitable conditions on the potential V (see (0.5) below), we show that, given $g \in L^p(\partial\Omega)$, $1 < p \leq 2$, there exists a unique solution u satisfying

$$(0.2) \quad \begin{cases} -\Delta u + Vu = 0 & \text{in } \Omega, \\ \frac{\partial u}{\partial N} = g & \text{a.e. on } \partial\Omega, \\ \|(\nabla u)^*\|_{L^p(\partial\Omega)} < \infty. \end{cases}$$

Here N denotes the outward unit normal to $\partial\Omega$ and $(\nabla u)^*$, the nontangential maximal function of ∇u , is defined by

$$(0.3) \quad (\nabla u)^*(Q) = \sup\{|\nabla u(X)| : X \in \Omega, |X - Q| < 2 \operatorname{dist}(X, \partial\Omega)\}$$