

On the Homogeneous Statistical Solutions of the 2-D Navier-Stokes Equations

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ABSTRACT. We show that the homogeneous statistical solutions of the Navier-Stokes equations constructed in [3], satisfy in the two dimensional case both the energy and the enstrophy inequalities.

Introduction. The purpose of this short note is to provide the proof for the remark made in [3] on the existence of homogeneous statistical solutions of the 2-D Navier-Stokes equations satisfying both the energy and the enstrophy inequalities; more specifically, we show that the homogeneous statistical solutions of the 2-D Navier-Stokes equations constructed by Foias and Temam in [3] satisfy the enstrophy inequality besides the energy inequality, if the initial data has finite mean enstrophy. It is fair to note that although the existence of statistical solutions for the Navier-Stokes equations was firstly done in [1] and [2], the much harder existence theorem for homogeneous statistical solutions is due to Vishik and Fursikov [5] (see also [6]). The proof in [3] uses as initial step the argument in [1]. Below we use the same notations and functional settings as given in Section 1 of [3] without reintroducing them here. We emphasize that throughout this note we consider only 2-D fluid flows.

Statement and proof of the results. Our purpose is to establish this:

Theorem 1. *Let there be given a homogeneous probability measure μ_0 on H_{loc} such that*

$$(1) \quad e(\mu_0) = \frac{1}{2} \int \frac{1}{|Q|} \int |u(x)|^2 d\mu_0(u) < \infty$$

and

$$(2) \quad \epsilon(\mu_0) = \frac{1}{2} \int \frac{1}{|Q|} \int_Q \left(\sum_{j=1}^2 |D_j u(x)|^2 \right) dx d\mu_0(u) < \infty.$$